DOI: 10.2478/auom-2019-0010
An. Şt. Univ. Ovidius Constanţa **Vol. 27(1),2019, 193–210**

Hyperideal theory in ordered Krasner hyperrings

Saber Omidi and Bijan Davvaz

Abstract

In this paper, we study some properties of ordered Krasner hyperrings. Also we state some definitions and basic facts and prove some results on ordered Krasner hyperring $(R, +, \cdot, \leq)$. In particular, we introduce the concepts of prime hyperideals and semiprime hyperideals of an ordered Krasner hyperring and present several examples of them.

1 Introduction and basic definitions

In [13], Heidari and Davvaz studied a semihypergroup $(H, \circ)$ besides a binary relation $\leq$, where $\leq$ is a partial order relation such that satisfies the monotone condition. Indeed, an *ordered semihypergroup* $(H, \circ, \leq)$ is a semihypergroup $(H, \circ)$ together with a partial order $\leq$ that is compatible with the hyperoperation, meaning that for any x, y, z in H,

$$x \leq y \Rightarrow z \circ x \leq z \circ y \text{ and } x \circ z \leq y \circ z.$$

Here, $z \circ x \leq z \circ y$ means for any $a \in z \circ x$ there exists $b \in z \circ y$ such that $a \leq b$. The case $x \circ z \leq y \circ z$ is defined similarly. The concept of ordered semihypergroups is a generalization of the concept of ordered semigroups. The concept of ordering hyperstructures introduced by Chvalina [7] as a special class of hypergroups and studied by many authors, for example, Bakhshi and Borzooei [5], Chvalina [7], Chvalina and Moucka [8], Davvaz et al. [6, 11], Ameri et

Key Words: Algebraic hyperstructure, Ordered Krasner hyperring, Prime hyperideal, Semiprime hyperideal, Convex hyperideal, M-system.
2010 Mathematics Subject Classification: Primary 16Y99; Secondary 20N20.
Received: 09.03.2018
Accepted: 29.05.2018

al. [1, 2], Hoskova [14, 15]. There are different types of hyperrings. If only the addition $+$ is a hyperoperation and the multiplication $\cdot$ is an operation, then we say that R is an additive hyperring. A special case of this type is the hyperring introduced by Krasner [17]. Some principal notions of hyperring theory can be found in [9, 12, 18, 19, 22, 23].

A *Krasner hyperring* [17] is an algebraic hypersructure $(R, +, \cdot)$ which satisfies the following axioms:

(1) $(R, +)$ is a canonical hypergroup [20], i.e., (i) for any $x, y, z \in R, x+(y+z) = (x+y)+z$, (ii) for any $x, y \in R, x+y = y+x$, (iii) there exists $0 \in R$ such that $0+x = x+0 = x$, for any $x \in R$, (iv) for every $x \in R$, there exists a unique element $x' \in R$, such that $0 \in x+x'$ (we shall write $-x$ for x' and we call it the opposite of x), (v) $z \in x+y$ implies that $y \in -x+z$ and $x \in z-y$, that is $(R, +)$ is reversible;

(2) $(R, \cdot)$ is a semigroup having zero as a bilaterally absorbing element, i.e., $x \cdot 0 = 0 \cdot x = 0$;

(3) The multiplication is distributive with respect to the hyperoperation $+$.

We call 0 the *zero* of the Krasner hyperring $(R, +, \cdot)$. For $x \in R$, let $-x$ denote the unique inverse of x in $(R, +)$. Then $-(-x) = x$, for all $x \in R$. In addition, we have $(x+y)\cdot(z+w) \subseteq x\cdot z+x\cdot w+y\cdot z+y\cdot w$, $(-x)\cdot y = x\cdot(-y) = -(x\cdot y)$, for all $x, y, z, w \in R$. A Krasner hyperring R is called *commutative* (with unit element) if $(R, \cdot)$ is a commutative semigroup (with unit element). A *Krasner hyperfield* is a Krasner hyperring for which $(R-\{0\}, \cdot)$ is a group. A non-empty subset I of a Krasner hyperring $(R, +, \cdot)$ is called a *left* (resp. *right*) *hyperideal* of R if $(I, +)$ is a canonical subhypergroup of $(R, +)$ and for every $a \in I$ and $r \in R$, $r \cdot a \in I$ (resp. $a \cdot r \in I$). A *hyperideal* of $(R, +, \cdot)$ is one which is a left as well as a right hyperideal of R. That is, $x+y \subseteq I$ and $-x \in I$, for all $x, y \in I$ and $x \cdot y, y \cdot x \in I$, for all $x \in I$ and $y \in R$. Let I be a hyperideal of R and $R/I = \{x+I \mid x \in R\}$. Define $(x+I)+(y+I) = \{(z+I) \mid z \in x+y\}$ and $(x+I)\cdot(y+I) = x\cdot y+I$, for all $x, y \in I$. Then $(R/I, +, \cdot)$ is a Krasner hyperring.

Now, we recall the following definition from [3]. A *partially ordered ring* is a ring $(R, +, \cdot)$, together with a compatible partial order, i.e., a partial order $\leq$ on the underlying set R that is compatible with the ring operations in the sense that it satisfies: (1) for all $a, b, c \in R$, $a \leq b$ implies that $a+c \leq b+c$; (2) for all $a, b \in R$, $0 \leq a$ and $0 \leq b$ we have $0 \leq a \cdot b$. An ordered ring, also called a *totally ordered ring*, is a partially ordered ring $(R, \leq)$ where $\leq$ is additionally a total order. An element $a \in R$ such that $0 \leq a$ is called *positive*. If P is the set of positive elements of a partially ordered ring, then $P+P \subseteq P$ and $P \cdot P \subseteq P$. Furthermore, $P \cap (-P) = \{0\}$. If R is an ordered ring, then the set $\{x : x \in R, x \geq 0\}$ is called the *positive cone*. The positive cone of an ordered ring completely defines the order $x \leq y$ if and only if $y - x \in P$. An *ordered field* is an ordered ring which is also a field. It is easy to see that if $a, b, c \in R$ with $a \leq b$ and $0 \leq c$, then $a \cdot c \leq b \cdot c$. Note that every ring is an ordered ring with the trivial order.

2 Hyperideals in ordered Krasner hyperrings

An algebraic hypersructure $(R, +, \cdot, \leq)$ is called an *ordered Krasner hyperring* if $(R, +, \cdot)$ is a Krasner hyperring with a partial order relation $\leq$ such that for all a, b and c in R:

(1) If $a \leq b$, then $a + c \leq b + c$, meaning that for any $x \in a + c$, there exists $y \in b + c$ such that $x \leq y$. The case $c + a \leq c + b$ is defined similarly.

(2) If $a \leq b$ and $0 \leq c$, then $a \cdot c \leq b \cdot c$ and $c \cdot a \leq c \cdot b$.

An element $a \in R$ is called *positive* if $0 \leq a$. The set of all positive elements of R is called the *positive cone* of R and is denoted by $P = R^+$. $x \in R$ is called *negative* if $x \leq 0$. The set of all negative elements of R is called the *negative cone* of R and is denoted by R^-.

Proposition 2.1. *In any ordered Krasner hyperring $(R, +, \cdot, \leq)$, for each $a, b \in R$, we have*

$$a \leq b \Leftrightarrow -b \leq -a.$$

Proof. For each $a, b \in R$, we have

$$\begin{aligned} a \leq b \quad &\Leftrightarrow (-a + b) \cap R^+ \neq \emptyset \\ &\Leftrightarrow (b - a) \cap R^+ \neq \emptyset \\ &\Leftrightarrow (a - b) \cap R^- \neq \emptyset \\ &\Leftrightarrow (-b + a) \cap R^- \neq \emptyset \\ &\Leftrightarrow -b \leq -a. \end{aligned}$$

□

EXAMPLE 1. Let $R = \{a, b, c\}$ be a set with the hyperoperation $\oplus$ and the binary operation $\odot$ defined as follows:

$\oplus$	a	b	c
a	a	b	c
b	b	b	R
c	c	R	c

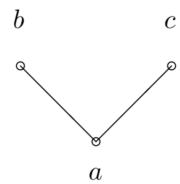
$\odot$	a	b	c
a	a	a	a
b	a	b	c
c	a	c	b

Then, $(R, \oplus, \odot)$ is a Krasner hyperring. We have $(R, \oplus, \odot, \leq)$ is an ordered Krasner hyperring where the order relation $\leq$ is defined by:

$$\leq := \{(a,a), (b,b), (c,c), (a,b), (a,c)\}.$$

The covering relation and the figure of R are given by:

$$\prec = \{(a,b), (a,c)\}.$$



Example 2. If $(H, \leq, +)$ is a totally ordered group, then

$$\begin{array}{l} x \oplus x = \{t \in H : t \leq x\} \text{ for all } x \in H, \\ x \oplus y = \{max\{x, y\}\} \text{ for all } x, y \in H, x \neq y, \end{array}$$

defines a structure of canonical hypergroup on H. If $(H, +, \cdot)$ is a totally ordered ring (for example $\mathbb{R}$), then $(H, \oplus, \cdot)$ is a Krasner hyperring [21]. Consider $(H, \oplus, \cdot)$ as a poset with the natural ordering. Then, $(H, \oplus, \cdot)$ is an ordered Krasner hyperring.

Example 3. Let $(R, +, \cdot)$ be a Krasner hyperring and $M(R) = \{(a,b) : a, b \in R\}$. The hyperoperation $\oplus$ and the multiplication $\odot$ are defined on $M(R)$ by:

$$\begin{array}{l} (a,b) \oplus (c,d) = \{(x,y) : x \in a + c, y \in b + d\}, \\ (a,b) \odot (c,d) = (ac, bd), \end{array}$$

for all $(a,b), (c,d) \in M(R)$. Clearly, this hyperoperation is well defined and $(M(R), \oplus)$ is a canonical hypergroup. The element $(0,0)$ is the additive identity of $M(R)$. Also, for each (a,b) of $M(R)$ there exists a unique element $(-a,-b) \in M(R)$ such that $(0,0) \in (a,b) \oplus (-a,-b)$. Also, the multiplication $\odot$ is well defined and associative. Therefore, $(M(R), \odot)$ is a semigroup. Now, let $(a,b), (c,d), (e,f) \in M(R)$. Then,

$$\begin{aligned} (a,b) \odot \Big((c,d) \oplus (e,f)\Big) &= (a,b) \odot \{(r,s) : r \in c + e, s \in d + f\} \\ &= \{(ar, bs) : r \in c + e, s \in d + f\} \end{aligned}$$

Also,

$$\begin{aligned} \Big((a,b) \odot (c,d)\Big) \oplus \Big((a,b) \odot (e,f)\Big) &= (ac, bd) \oplus (ae, bf) \\ &= \{(g,h) : g \in ac + ae, h \in bd + bf\}. \end{aligned}$$

By the left distributive axiom of R,

$$(a,b)\odot\Big((c,d)\oplus(e,f)\Big)=\Big((a,b)\odot(c,d)\Big)\oplus\Big((a,b)\odot(e,f)\Big).$$

Similarly, we can show that the right distributive law is also satisfied on $M(R)$. Thus, $(M(R),\oplus,\odot)$ is a Krasner hyperring. Now, let $(R,+,\cdot,\leq)$ be an ordered Krasner hyperring. Define the order relation $\preceq$ on $M(R)$ by:

$$\big(a,b\big)\preceq\big(a',b'\big)\Leftrightarrow a\leq a', b\leq b'$$

Then, $(M(R),\oplus,\odot,\preceq)$ is an ordered Krasner hyperring.

A *homomorphism* from an ordered Krasner hyperring $(R_1,+_1,\cdot_1,\leq_1)$ into an ordered Krasner hyperring $(R_2,+_2,\cdot_2,\leq_2)$ is a function $\varphi: R_1\to R_2$ such that (1) $\varphi(a+_1 b)\subseteq\varphi(a)+_2\varphi(b)$; (2) $\varphi(a\cdot_1 b)=\varphi(a)\cdot_2\varphi(b)$; (3) If $a\leq_1 b$, then $\varphi(a)\leq_2\varphi(b)$. Also φ is called a good (strong) homomorphism if in the previous condition (1), the equality is valid. An *isomorphism* from $(R_1,+_1,\cdot_1,\leq_1)$ into $(R_2,+_2,\cdot_2,\leq_2)$ is a bijective good homomorphism from $(R_1,+_1,\cdot_1,\leq_1)$ onto $(R_2,+_2,\cdot_2,\leq_2)$. The kernel of φ, $ker\varphi$, is defined by $ker\varphi=\{x\in R_1\mid\varphi(x)=0_2\}$, where 0_2 is the zero of $(R_2,+_2,\cdot_2)$. If R_1 is isomorphic to R_2, then it is denoted by $R_1\cong R_2$.

Let $(R,+,\cdot,\leq)$ be an ordered Krasner hyperring. A subset I of R is called a *hyperideal* of R if it satisfies the following conditions: (1) $(I,+)$ is a canonical subhypergroup of $(R,+)$; (2) $x\cdot y\in I$ and $y\cdot x\in I$ for all $x\in I$ and $y\in R$; (3) When $x\in I$ and $y\in R$ such that $y\leq x$, imply that $y\in I$.

Let φ be a homomorphism from an ordered Krasner hyperring R_1 into an ordered Krasner hyperring R_2. Then, $ker\varphi$ is a hyperideal of R_1 and $Im\varphi$ is a subhyperring of R_2. In [10], Davvaz gave the fundamental homomorphism theorem of Krasner hyperrings. Now, we drive this theorem in the context of hyperrings.

Theorem 2.2. *Let φ be a homomorphism from an ordered Krasner hyperring R into an ordered Krasner hyperring T. Define $\theta: R/ker\varphi\to T$ by $\theta(x+ker\varphi)=\varphi(x)$, for all $x\in R$. Then, the following statements hold.*

(1) θ is a homomorphism from $R/ker\varphi$ onto T.

(2) If φ is a good (strong) homomorphism, then θ is an isomorphism and hence $R/ker\varphi\cong T$.

Proof. (1) We check the conditions of definition. Let $x,y\in R$ be such that $x+ker\varphi=y+ker\varphi$. Then, $x\in y+ker\varphi$, so $x\in y+z$ for some $z\in ker\varphi$.

Thus, $\varphi(x) \in \varphi(y+z) \subseteq \varphi(y)+\varphi(z) = \varphi(y)+0 = \{\varphi(y)\}$. So, $\varphi(x) = \varphi(y)$. Thus, the map θ is well-defined. If $x, y \in R$, then we have

$$\begin{aligned}&\theta((x+ker\varphi)+(y+ker\varphi)) = \theta(\{z+ker\varphi : z \in x+y\})\\ &= \{\theta(z+ker\varphi) : z \in x+y\} = \{\varphi(z) : z \in x+y\}\end{aligned}$$

Also,

$$\begin{aligned}&\theta(x+ker\varphi)+\theta(y+ker\varphi) = \varphi(x)+\varphi(y)\\ &\supseteq \varphi(x+y) = \{\varphi(z) : z \in x+y\}\end{aligned}$$

Thus, $\theta((x+ker\varphi)+(y+ker\varphi)) \subseteq \theta(x+ker\varphi)+\theta(y+ker\varphi)$. So, the first condition of definition is verified. We have

$$\begin{aligned}&\theta(x+ker\varphi)(y+ker\varphi) = \theta(xy+ker\varphi) = \varphi(xy)\\ &= \varphi(x)\varphi(y) = \theta(x+ker\varphi)+\theta(y+ker\varphi).\end{aligned}$$

So, the second condition of definition is verified. Now, let $x \leq_R y$. Since φ is a homomorphism, we have $\varphi(x) \leq_T \varphi(y)$. Thus $\theta(x+ker\varphi) \leq_T \theta(y+ker\varphi)$. So, the third condition of definition is verified. Therefore, θ is a homomorphism.

(2) Assume that φ is a good (strong) homomorphism. It can be seen from the proof of (1), that θ is a good (strong) homomorphism. We know that $0+ker\varphi \in ker\theta$. Let $x \in R$ be such that $\theta(x+ker\varphi) = 0$. Then $\varphi(x) = 0$, so $x \in ker\varphi$. Hence $x+ker\varphi = 0+ker\varphi$. Thus we have $ker\theta = \{0+ker\varphi\}$. Hence θ is one to one. Clearly, θ is onto. Thus θ is a good (strong) isomorphism. That is $R/ker\varphi$ is strongly isomorphic to T. □

Theorem 2.3. *Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring with positive cone P and $\varphi : R \to R$ be any good (strong) homomorphism of the canonical hypergroup $(R, +)$ such that $\varphi(P) \subseteq P$. Assume that for any $r \in R$, there exists an integer $n \geq 1$ such that $\varphi^n(r) = r$. Then, φ is the identity map.*

Proof. If $a < b$, then $b - a \subseteq P$. So, by hypothesis $\varphi(b-a) \subseteq P$. Since φ is a good (strong) homomorphism of $(R, +)$, it follows that $\varphi(b-a) = \varphi(b)-\varphi(a)$. Therefore, $\varphi(a) < \varphi(b)$. Now, let $\varphi \neq id$. Then $\varphi(r) \neq r$ for some $r \in R$. We have either $r < \varphi(r)$ or $\varphi(r) < r$. Say $r < \varphi(r)$. Fix an integer $n \geq 1$ such that $\varphi^n(r) = r$. Then, we have

$$r < \varphi(r) < \varphi^2(r) < \cdots < \varphi^n(r) = r$$

a contradiction. If $\varphi(r) < r$, a similar contradiction results. Therefore, φ is the identity map. □

In the following, we shall specialize our study to some of the basic facts concerning ordered Krasner hyperrings.

Definition 2.4. A non-empty subset P of an ordered Krasner hyperring $(R, +, \cdot, \leq)$ is called a *prime hyperideal* of R if the following conditions hold:

(1) $A \cdot B \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$ for any two hyperideal A and B of R.

(2) If $x \in P$ and $y \leq x$, then $y \in P$ for every $y \in R$.

EXAMPLE 4. Define the hyperoperation $\oplus$ and the operation $\odot$ on the set $R = \{0, 1\}$ by

$\oplus$	0	1
0	0	1
1	1	$\{0, 1\}$

$\odot$	0	1
0	0	0
1	0	1

Then, $(R, \oplus, \odot)$ is a commutative Krasner hyperring with the zero element 0. Consider $(R, \oplus, \odot)$ as a poset with the natural ordering. Thus, $(R, \oplus, \odot)$ is an ordered Krasner hyperring. Now, it is easy to see that $\{0\}$ and $\{0, 1\}$ are hyperideals of R. It is obvious that $\{0\}$ is a prime hyperideal of R.

EXAMPLE 5. Consider the hyperring $R = \{0, a, b\}$ with the hyperaddition $\oplus$ and the multiplication $\odot$ defined as follows:

$\oplus$	0	a	b
0	0	a	b
a	a	$\{a, b\}$	R
b	b	R	$\{a, b\}$

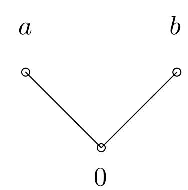
$\odot$	0	a	b
0	0	0	0
a	0	b	a
b	0	a	b

Then, $(R, \oplus, \odot)$ is a Krasner hyperring [4]. We have $(R, \oplus, \odot, \leq)$ is an ordered Krasner hyperring, where the order relation $\leq$ is defined by:

$$\leq := \{(0, 0), (a, a), (b, b), (0, a), (0, b)\}.$$

The covering relation and the figure of R are given by:

$$\prec = \{(0, a), (0, b)\}.$$



Now, it is easy to see that $\{0\}$ and $\{0, a, b\}$ are hyperideals of R. It is obvious that $\{0\}$ is a prime hyperideal of R.

Example 6. Let $R = \{0, a, b, c\}$ be a set with the hyperoperation $\oplus$ and the multiplication $\odot$ defined as follows:

$\oplus$	0	a	b	c
0	0	a	b	c
a	a	$\{0, b\}$	$\{a, c\}$	b
b	b	$\{a, c\}$	$\{0, b\}$	a
c	c	b	a	0

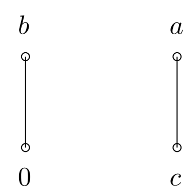
$\odot$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	b	0
c	0	c	0	c

Then, $(R, \oplus, \odot)$ is a Krasner hyperring [4]. We have $(R, \oplus, \odot, \leq)$ is an ordered Krasner hyperring where the order relation $\leq$ is defined by:

$$\leq := \{(0, 0), (a, a), (b, b), (c, c), (0, b), (c, a)\}.$$

The covering relation and the figure of R are given by:

$$\prec = \{(0, b), (c, a)\}.$$



Now, it is easy to see that $I_1 = \{0\}$, $I_2 = \{0, b\}$, $I_3 = \{0, c\}$, $I_4 = \{0, b, c\}$ and $I_5 = \{0, a, b, c\}$ are hyperideals of R. Also I_2, I_3 and I_4 are prime hyperideals of R. The hyperideal $I_1 = \{0\}$ is not a prime hyperideal of R. Indeed, $\{0, b\} \odot \{0, c\} = \{0\}$, but $\{0, b\} \nsubseteq \{0\}$ and $\{0, c\} \nsubseteq \{0\}$.

Example 7. Let $R = \{a, b, c, d, e, f\}$ be a set with the hyperoperation $\oplus$ and the multiplication $\odot$ defined as follows:

$\oplus$	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	$\{a, b\}$	d	$\{c, d\}$	f	$\{e, f\}$
c	c	d	c	d	$\{a, c, e\}$	$\{b, d, f\}$
d	d	$\{c, d\}$	d	$\{c, d\}$	$\{b, d, f\}$	R
e	e	f	$\{a, c, e\}$	$\{b, d, f\}$	e	f
f	f	$\{e, f\}$	$\{b, d, f\}$	R	f	$\{e, f\}$

and

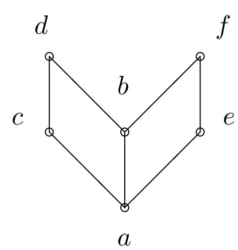
$\odot$	a	b	c	d	e	f
a	a	a	a	a	a	a
b	a	b	a	b	a	b
c	a	a	c	c	e	e
d	a	b	c	d	e	f
e	a	a	e	e	c	c
f	a	b	e	f	c	d

Then, $(R, \oplus, \odot)$ is a Krasner hyperring. We have $(R, \oplus, \odot, \leq)$ is an ordered Krasner hyperring, where the order relation $\leq$ is defined by:

$$\begin{aligned} \leq \quad := \{&(a,a),(b,b),(c,c),(d,d),(e,e),(f,f),(a,b),(a,c),\\ &(a,d),(a,e),(a,f),(b,d),(b,f),(c,d),(e,f)\}. \end{aligned}$$

The covering relation and the figure of R are given by:

$$\prec = \{(a,b),(a,c),(a,e),(b,d),(b,f),(c,d),(e,f)\}.$$



It is easy to see that $\{a\}$, $\{a,b\}$, $\{a,c,e\}$ and $\{a,b,c,d,e,f\}$ are hyperideals of R. It is obvious that $\{a,b\}$ and $\{a,c,e\}$ are prime hyperideals of R. The hyperideal $\{a\}$ is not a prime hyperideal of R. Indeed, $\{a,b\} \odot \{a,c,e\} = \{a\}$, but $\{a,b\} \nsubseteq \{a\}$ and $\{a,c,e\} \nsubseteq \{a\}$.

Definition 2.5. A non-empty subset I of an ordered Krasner hyperring $(R, +, \cdot, \leq)$ is called a *semiprime hyperideal* of R if the following conditions hold:

(1) $A \cdot A \subseteq I$ implies that $A \subseteq I$ for any hyperideal A of R.

(2) If $x \in I$ and $y \leq x$, then $y \in I$ for every $y \in R$.

Remark 1. Every prime hyperideal of R is a semiprime hyperideal of R.

EXAMPLE 8. In Example 6, $I_1 = \{0\}$ is a semiprime hyperideal, but is not a prime hyperideal.

Definition 2.6. An ordered Krasner hyperring $(R, +, \cdot, \leq)$ is said to be a *prime hyperring* if $a \cdot R \cdot b = 0$ for $a, b \in R$ implies either $a = 0$ or $b = 0$. Equivalently, an ordered Krasner hyperring R is called prime if $a \cdot r \cdot b = 0$ for all $r \in R$ implies either $a = 0$ or $b = 0$.

EXAMPLE 9. In Example 4 and Example 5, R is prime, but in Example 6 and Example 7, R is not prime.

Definition 2.7. An ordered Krasner hyperring $(R, +, \cdot, \leq)$ is said to be a *semiprime hyperring* if $a \cdot R \cdot a = 0$ for $a \in R$ implies $a = 0$. Equivalently, an ordered Krasner hyperring R is called semiprime if $a \cdot r \cdot a = 0$ for all $r \in R$ implies $a = 0$.

REMARK 2. Every prime ordered Krasner hyperring is a semiprime ordered Krasner hyperring.

EXAMPLE 10. In Example 6, R is a semiprime ordered Krasner hyperring, but is not a prime ordered Krasner hyperring.

Definition 2.8. Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring with the positive cone P. A subset $A \subseteq R$ is *convex* if $0 \leq p \leq q$, $q \in A$ implies that $p \in A$. Equivalently, A is convex if $p_1 + p_2 \subseteq A$, $p_i \in P$, implies that $p_i \in A$, $i = 1, 2$. A hyperideal A of an ordered Krasner hyperring $(R, +, \cdot, \leq)$ is said to be convex if it is convex as a subset.

EXAMPLE 11. (1) In Example 4, the hyperideals $\{0\}$ and $\{0, 1\}$ are convex.

(2) In Example 5, the hyperideals $\{0\}$ and $\{0, a, b\}$ are convex.

(3) In Example 6, the hyperideals I_1, I_2, I_3, I_4 and I_5 are convex.

(4) In Example 7, the hyperideals $\{a\}$, $\{a, b\}$, $\{a, c, e\}$ and $\{a, b, c, d, e, f\}$ are convex.

Theorem 2.9. *Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. A hyperideal $I \subseteq R$ is the kernel of a homomorphism in an ordered Krasner hyperring if and only if I is a convex hyperideal of R.*

Proof. Let $\varphi : R \to R$ be a homomorphism and $I = ker\varphi$. Let $0 \leq p \leq q$ in R with $\varphi(q) = 0$. Since φ is a homomorphism, it follows that $0 \leq \varphi(p) \leq \varphi(q) = 0$ in R. Thus we have $\varphi(p) = 0$. Therefore, $I = ker\varphi$ is a convex hyperideal of R.

Conversely, suppose that I is a convex hyperideal of R. Consider the projection map $\pi : R \to R/I$. We can impose an order on R/I so that π is order preserving if, whenever $\sum\limits_{i=1}^{n} p_i a_i^2 \subseteq I$, $p_i \in P$, $a_i \in R$, then $p_j a_j^2 \in I$, $1 \leq j \leq n$. This is the second characterization of convexity of Definition 2.8. The weakest order on R/I such that π is order preserving, namely $\pi_*(P) = \{p + I : p \in P\}$, will be called the induced order. □

Definition 2.10. A convex hyperideal $Q \subset R$ is a *maximal convex hyperideal* if $Q \neq R$ and whenever $Q \subseteq Q'$, Q' a convex hyperideal, either $Q' = Q$ or $Q' = R$.

Now, we establish the existence of maximal convex hyperideals.

Theorem 2.11. *Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. Let $I \subsetneqq R$ be a convex hyperideal. Then, I is contained in at least one maximal convex hyperideal.*

Proof. The family of all convex hyperideals containing I but not containing 1 is non-empty, partially ordered by inclusion, and satisfies the chain condition. Thus by Zorn's Lemma the proof completes. □

Remark 3. Since $I = (0)$ is always a convex hyperideal of R, we conclude that any non-zero ordered Krasner hyperring $(R, +, \cdot, \leq)$ has maximal convex hyperideal.

Remark 4. Maximal convex hyperideals are prime.

Definition 2.12. If $(R, +, \cdot, \leq)$ is an ordered Krasner hyperring and $A \subseteq R$, then $(A]$ is the subset of R defined as follows:

$$(A] = \{t \in R : t \leq a, \text{ for some } a \in A\}.$$

Lemma 2.13. *Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. If A and B are non-empty subsets of R, then we have:*

(1) $A \subseteq (A]$;

(2) *If* $A \subseteq B$, *then* $(A] \subseteq (B]$;

(3) $((A]] = (A]$;

(4) $(A \cup B] = (A] \cup (B]$;

(5) $(A] + (B] \subseteq (A + B]$;

(6) $(A] \cdot (B] \subseteq (A \cdot B]$;

(7) $((A] \cdot (B]] = (A \cdot B]$;

(8) *If* $A, B, C \subseteq R$ *such that* $A \subseteq B$, *then* $A+C \subseteq B+C$ *and* $C+A \subseteq C+B$;

(9) *If* $A, B, C \subseteq R$ *such that* $A \subseteq B$, *then* $A \cdot C \subseteq B \cdot C$ *and* $C \cdot A \subseteq C \cdot B$.

Proof. The proof is straightforward. □

Definition 2.14. Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. A non-empty subset S of R is called an *M-system* of R if for each $a, b \in S$, there exist $r \in R$ and $c \in S$ such that $c \leq a \cdot (r \cdot b)$ or equivalently $c \in (a \cdot (R \cdot b)]$.

EXAMPLE 12. (1) The set $\{0, 1\}$ is an M-system of an ordered Krasner hyperring defined in Example 4.

(2) The sets $\{0, a\}$, $\{0, b\}$, $\{a, b\}$ and $\{0, a, b\}$ are an M-system of an ordered Krasner hyperring defined in Example 5.

(3) The sets $\{0, a\}$, $\{0, b\}$, $\{0, c\}$, $\{a, b\}$ and $\{a, c\}$ are an M-system of an ordered Krasner hyperring defined in Example 6, but $\{b, c\}$ is not an M-system of an ordered Krasner hyperring defined in Example 6.

(4) The sets $\{a\}$, $\{a, b\}$ and $\{a, c, e\}$ are an M-system of an ordered Krasner hyperring defined in Example 7, but $\{b, c\}$ is not an M-system of an ordered Krasner hyperring defined in Example 7.

Definition 2.15. Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. A non-empty subset S of R is called an *N-system* of R if for each $a \in S$, there exist $r \in R$ and $c \in S$ such that $c \leq a \cdot (r \cdot a)$ or equivalently $c \in (a \cdot (R \cdot a)]$.

REMARK 5. Every M-system of R is an N-system of R.

EXAMPLE 13. The set $\{b, c\}$ is an N-system of an ordered Krasner hyperring defined in Example 6, but is not an M-system of an ordered Krasner hyperring defined in Example 6.

Definition 2.16. A non-empty subset I of an ordered Krasner hyperring $(R, +, \cdot, \leq)$ is called a *quasi-prime hyperideal* of R if for all left hyperideals A, B of R, $A \cdot B \subseteq I$ implies that $A \subseteq I$ or $B \subseteq I$.

Definition 2.17. A non-empty subset I of an ordered Krasner hyperring $(R, +, \cdot, \leq)$ is called a *quasi-semiprime hyperideal* of R if for any left hyperideal A of R, $A \cdot A \subseteq I$ implies that $A \subseteq I$.

REMARK 6. Every quasi-prime hyperideal of R is a quasi-semiprime hyperideal of R.

EXAMPLE 14. In Example 6, $\{0\}$ is a quasi-semiprime hyperideal of R, but is not a quasi-prime hyperideal of R.

Definition 2.18. A non-empty subset I of an ordered Krasner hyperring $(R, +, \cdot, \leq)$ is called a *quasi-irreducible hyperideal* of R if for all left hyperideals A, B of R, $A \cap B \subseteq I$ implies that $A \subseteq I$ or $B \subseteq I$.

EXAMPLE 15. In Example 6, $\{0, b\}$, $\{0, c\}$ and $\{0, b, c\}$ are quasi-irreducible hyperideals of R, but $\{0\}$ is not a quasi-irreducible hyperideal of R.

Lemma 2.19. *Let I be a left hyperideal of an ordered Krasner hyperring $(R, +, \cdot, \leq)$. Then, I is quasi-prime hyperideal if and only if for all $a, b \in R$, $a \cdot (R \cdot b) \subseteq I$ implies that $a \in I$ or $b \in I$.*

Proof. It is straightforward. □

Theorem 2.20. *Let I be a left hyperideal of an ordered Krasner hyperring $(R, +, \cdot, \leq)$. Then, I is quasi-prime hyperideal if and only if $R \setminus I$ is an M-system.*

Proof. Let I be a quasi-prime hyperideal and $a, b \in R \setminus I$. Assume that $c \notin (a \cdot (R \cdot b)]$ for all $c \in R \setminus I$. Then $(a \cdot (R \cdot b)] \subseteq I$. This implies that $a \cdot (R \cdot b) \subseteq I$. So, $a \in I$ or $b \in I$, which contradicts the assumption that $a, b \in R \setminus I$. Hence $R \setminus I$ is an M-system.

Conversely, let $R \setminus I$ be an M-system and $a \cdot (R \cdot b) \subseteq I$ for some $a, b \in R \setminus I$. Then there exist $c \in R \setminus I$ and $x \in R$ such that $c \leq a \cdot (x \cdot b)$, which implies that $c \in I$, it contradicts the assumption $c \in R \setminus I$. Hence I is a quasi-prime hyperideal of R. □

Lemma 2.21. *Let I be a left hyperideal of an ordered Krasner hyperring $(R, +, \cdot, \leq)$. Then, I is quasi-semiprime hyperideal if and only if for all $a \in R$, $a \cdot (R \cdot a) \subseteq I$ implies that $a \in I$.*

Proof. It is straightforward. □

Theorem 2.22. *Let I be a left hyperideal of an ordered Krasner hyperring $(R, +, \cdot, \leq)$. Then, I is quasi-semiprime hyperideal if and only if $R \setminus I$ is an N-system.*

Proof. Let I be a quasi-semiprime hyperideal and $a \in R \setminus I$. Assume that $c \notin (a \cdot (R \cdot a)]$ for all $c \in R \setminus I$. Then $(a \cdot (R \cdot a)] \subseteq I$. This implies that $a \cdot (R \cdot a) \subseteq I$. So, $a \in I$, which contradicts the assumption that $a \in R \setminus I$. Hence $R \setminus I$ is an N-system.

Conversely, let $R \setminus I$ be an N-system and $a \cdot (R \cdot a) \subseteq I$ with $a \notin I$. Then there exist $c \in R \setminus I$ and $r \in R$ such that $c \leq a \cdot (r \cdot a)$, which implies that $c \in I$, it contradicts the assumption $c \in R \setminus I$. Hence $a \in I$. Therefore, I is a quasi-semiprime hyperideal of R. □

Theorem 2.23. *If N is an N-system of an ordered Krasner hyperring$(R, +, \cdot, \leq)$ and $a \in N$, then there exists an M-system M of R such that $a \in M \subseteq N$.*

Proof. Let N be an N-system of an ordered Krasner hyperring R and $a \in N$. Then, by definition of N-system, there exist some $c_1 \in N$ such that $c_1 \in (a \cdot (R \cdot a)]$, so $(a \cdot (R \cdot a)] \cap N \neq \emptyset$. Take $a_1 \in (a \cdot (R \cdot a)] \cap N$ and again using the definition of N-system, there exist $c_2 \in N$ such that $c_2 \in (a_1 \cdot (R \cdot a_1)]$, so $(a_1 \cdot (R \cdot a_1)] \cap N \neq \emptyset$. Continuing in this way, we take $a_i \in (a_{i-1} \cdot (R \cdot a_{i-1})] \cap N \neq \emptyset$. Take $a_0 = a$ and define $M = \{a_0, a_1, \cdots\}$. Then, M is an M-system and $a \in M \subseteq N$. $\square$

Now, we recall the definition of a regular ring. An element a in a ring R is said to be regular if $a \in aRa$. A ring R is called regular if every element of R is regular. In the following, we present some results on regular ordered Krasner hyperrings.

Definition 2.24. Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. An element $a \in R$ is said to be *regular* if there exists an element $x \in R$ such that $a \leq (a \cdot x) \cdot a$. An ordered Krasner hyperring $(R, +, \cdot, \leq)$ is said to be regular if every element of R is regular.

Example 16. The ordered Krasner hyperring $(R, \oplus, \odot)$ defined as in Example 4, is regular.

Definition 2.25. Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. An element $a \in R$ is said to be *right regular* if $a \in (a^2 \cdot R]$.

Theorem 2.26. *Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. Then, R is a regular ordered Krasner hyperring if and only if $(A \cdot B] = (A \cap B]$ for right hyperideal A and left hyperideal B of R.*

Proof. Let R be regular. It is clear that $(A \cdot B] \subseteq (A \cap B]$. If $c \in (A \cap B]$, then $c \leq z$ for some $z \in A \cap B$. Since R is regular, there exists an element $x \in R$ such that $c \leq (c \cdot x) \cdot c$. We have $c \leq (c \cdot x) \cdot c \subseteq (c \cdot x) \cdot z \subseteq ((A \cdot R) \cdot B]$. Thus $c \in ((A \cdot R) \cdot B] \subseteq (A \cdot B]$. Hence, $(A \cap B] \subseteq (A \cdot B]$. Therefore, we have $(A \cdot B] = (A \cap B]$.

Conversely, let $a \in R$. Then we have $a \in (a \cdot R] \cap (R \cdot a] = ((a \cdot R) \cdot (R \cdot a)] = (a \cdot R \cdot a]$. So, there exists an element $x \in R$ such that $a \leq (a \cdot x) \cdot a$. Therefore, R is a regular ordered Krasner hyperring. $\square$

Theorem 2.27. *Every hyperideal of a regular ordered Krasner hyperring R is a prime hyperideal if and only if it is an irreducible hyperideal of R.*

Proof. Suppose that P is prime hyperideal of R and $(A \cap B] \subseteq P$. By Theorem 2.26, $(A \cdot B] = (A \cap B]$, so $(A \cdot B] \subseteq P$ which implies that $(A] \subseteq P$ or $(B] \subseteq P$.

Therefore, P is irreducible hyperideal of R.

Conversely, suppose that P is an irreducible hyperideal of R. Then $(A \cap B] \subseteq P$ implies that $(A] \subseteq P$ or $(B] \subseteq P$. By Theorem 2.26, $(A \cdot B] = (A \cap B]$, and so P is a prime hyperideal of R. $\square$

Definition 2.28. An ordered Krasner hyperring $(R, +, \cdot, \leq)$ is called *intra-regular* if for every $a \in R$, there exists $x, y \in R$ such that $a \leq x \cdot a^2 \cdot y$, or equivalently $a \in (R \cdot a^2 \cdot R]$.

Example 17. The ordered Krasner hyperring $(R, \oplus, \odot)$ defined as in Example 4, is intra-regular.

The notion of pseudoorder on an ordered semigroup was introduced and studied by Kehayopulu and Tsingelis [16]. Now, we continue this section with a similar definition for ordered Krasner hyperrings.

Definition 2.29. Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. A relation ρ on R is called *pseudoorder* if the following conditions hold:

(1) $\leq \subseteq \rho$;

(2) $a\rho b$ and $b\rho c$ imply $a\rho c$;

(3) $a\rho b$ implies $a + c\overline{\overline{\rho}}b + c$ and $c + a\overline{\overline{\rho}}c + b$, for all $c \in R$;

(4) $a\rho b$ implies $a \cdot c\overline{\overline{\rho}}b \cdot c$ and $c \cdot a\overline{\overline{\rho}}c \cdot b$, for all $c \in R$.

Definition 2.30. Let $(R, +, \cdot, \leq_R)$ and $(T, \oplus, \otimes, \leq_T)$ be two ordered Krasner hyperrings. Under the coordinatewise multiplication, i.e.,

$$(r_1, t_1) \boxplus (r_2, t_2) = (r_1 + r_2, t_1 \oplus t_2),$$
$$(r_1, t_1) \star (r_2, t_2) = (r_1 \cdot r_2, t_1 \otimes t_2),$$

where $(r_1, t_1), (r_2, t_2) \in R \times T$, the Cartesian product $R \times T$ of R and T forms a Krasner hyperring. Define a partial order $\leq$ on $R \times T$ by $(r_1, t_1) \leq (r_2, t_2)$ if and only if $r_1 \leq_R r_2$ and $t_1 \leq_T t_2$, where $(r_1, t_1), (r_2, t_2) \in R \times T$. Then, $(R \times T, \boxplus, \star, \leq)$ is an ordered Krasner hyperring.

Definition 2.31. Let $(R, +, \cdot, \leq_R)$ and $(T, \oplus, \otimes, \leq_T)$ be two ordered Krasner hyperrings, ρ_1, ρ_2 be two pseudoorders on R, T, respectively. On $R \times T$ we define:

$$(r_1, t_1)\rho(r_2, t_2) \Leftrightarrow r_1\rho_1 r_2 \text{ and } t_1\rho_2 t_2.$$

Lemma 2.32. *In Definition 2.31, ρ is pseudoorder on $R \times T$.*

Proof. It is straightforward. $\square$

Theorem 2.33. *Let $(R,+,\cdot,\leq_R)$ and $(T,\oplus,\otimes,\leq_T)$ be two ordered Krasner hyperrings, ρ_1, ρ_2 be two pseudoorders on R, T, respectively. Then,*

$$(R\times T)/\rho^* \cong R/\rho_1^* \times T/\rho_2^*.$$

Proof. We consider the map $\psi:(R\times T)/\rho^* \to R/\rho_1^*\times T/\rho_2^*$ by $\psi(\rho^*(r,t)) = (\rho_1^*(r),\rho_2^*(t))$. Suppose that $\rho^*(r_1,t_1) = \rho^*(r_2,t_2)$. Then, $(r_1,t_1)\rho^*(r_2,t_2)$ which implies that $(r_1,t_1)\rho(r_2,t_2)$ and $(r_2,t_2)\rho(r_1,t_1)$. Hence, $r_1\rho_1 r_2$, $t_1\rho_2 t_2$, $r_2\rho_1 r_1$ and $t_2\rho_2 t_1$ which imply that $r_1\rho_1^* r_2$ and $t_1\rho_2^* t_2$. So, $(\rho_1^*(r_1),\rho_2^*(t_1)) = (\rho_1^*(r_2),\rho_2^*(t_2))$. This means that $\psi(\rho^*(r_1,t_1)) = \psi(\rho^*(r_2,t_2))$. Therefore, ψ is well defined. Now, we show that ψ is a homomorphism. Suppose that $\rho^*(r_1,t_1)$ and $\rho^*(r_2,t_2)$ are two arbitrary elements of $(R\times T)/\rho^*$. Then,

$$\begin{aligned}
\psi(\rho^*(r_1,t_1)\uplus\rho^*(r_2,t_2)) &= \psi(\rho^*(r,t)),\ \text{for all } (r,t)\in(r_1,t_1)\boxplus(r_2,t_2)\\
&= (\rho_1^*(r),\rho_2^*(t)),\ \text{for all } r\in r_1+r_2,\ t\in t_1\oplus t_2\\
&= (\rho_1^*(r_1)+\rho_1^*(r_2),\rho_2^*(t_1)\oplus\rho_2^*(t_2))\\
&= (\rho_1^*(r_1),\rho_2^*(t_1))\boxplus(\rho_1^*(r_2),\rho_2^*(t_2))\\
&= \psi(\rho^*(r_1,t_1))\boxplus\psi(\rho^*(r_2,t_2)).
\end{aligned}$$

So, the first condition of the definition of homomorphism is verified. Suppose that $\rho^*(r_1,t_1)$ and $\rho^*(r_2,t_2)$ are two arbitrary elements of $(R\times T)/\rho^*$. Then,

$$\begin{aligned}
\psi(\rho^*(r_1,t_1)\triangledown\rho^*(r_2,t_2)) &= \psi(\rho^*(r,t)),\ \text{for } (r,t)=(r_1,t_1)\star(r_2,t_2)\\
&= (\rho_1^*(r),\rho_2^*(t)),\ \text{for } r=r_1\cdot r_2,\ t=t_1\otimes t_2\\
&= (\rho_1^*(r_1)\odot\rho_1^*(r_2),\rho_2^*(t_1)\diamond\rho_2^*(t_2))\\
&= (\rho_1^*(r_1),\rho_2^*(t_1))\times(\rho_1^*(r_2),\rho_2^*(t_2))\\
&= \psi(\rho^*(r_1,t_1))\times\psi(\rho^*(r_2,t_2)).
\end{aligned}$$

So, the second condition of the definition of homomorphism is verified. Now, suppose that $\rho^*(r_1,t_1)\preceq\rho^*(r_2,t_2)$. Then, $(r_1,t_1)\rho(r_2,t_2)$ which implies that $r_1\rho_1 r_2$ and $t_1\rho_2 t_2$. Thus, $\rho_1^*(r_1)\preceq_R\rho_1^*(r_2)$ and $\rho_2^*(t_1)\preceq_T\rho_2^*(t_2)$. Hence, $(\rho_1^*(r_1),\rho_2^*(t_1))\preceq_{R\times T}(\rho_1^*(r_2),\rho_2^*(t_2))$. This means that $\psi(\rho^*(r_1,t_1))\preceq_{R\times T}\psi(\rho^*(r_2,t_2))$, and so the third condition of the definition of homomorphism is verified. Therefore, ψ is a homomorphism. Clearly, ψ is onto. So, we show that it is one to one. Suppose that $\psi(\rho^*(r_1,t_1)) = \psi(\rho^*(r_2,t_2))$. Then, $(\rho_1^*(r_1),\rho_2^*(t_1)) = (\rho_1^*(r_2),\rho_2^*(t_2))$ and so $\rho_1^*(r_1)=\rho_1^*(r_2)$ and $\rho_2^*(t_1)=\rho_2^*(t_2)$. Hence, $(r_1,r_2)\in\rho_1^*$ and $(t_1,t_2)\in\rho_2^*$. This implies that $r_1\rho_1 r_2$, $r_2\rho_1 r_1$, $t_1\rho_2 t_2$ and $t_2\rho_2 t_1$. Thus, $(r_1,t_1)\rho(r_2,t_2)$ and $(r_2,t_2)\rho(r_1,t_1)$. Therefore, $(r_1,t_1)\rho^*(r_2,t_2)$ or $\rho^*(r_1,t_1)=\rho^*(r_2,t_2)$. Therefore, ψ is an isomorphism and so the proof is completed. □

Open problem. What is a necessary and sufficient condition for a Krasner hyperring $(R,+,\cdot)$ to be orderable?

References

[1] R. Ameri, A. Kordi and S. Hoskova-Mayerova, *Multiplicative hyperring of fractions and coprime hyperideals*, An. Stiint. Univ. "Ovidius" Constanta Ser. Mat., **25(1)** (2017), 5-23.

[2] R. Ameri, S. Hoskova-Mayerova and A. Kordi, *Pseudo regular rings derived from multiplicative hyperrings*, In: Aplimat 16th Conference on Applied Mathematics 2017 Proceedings. Bratislava: Vydavatelstvo STU Slovak University of Technology in Bratislava, 2017, pp. 17-27.

[3] F. W. Anderson, *Lattice-ordered rings of quotients*, Canadian Journal of Mathematics, **17** (1965), 434-448.

[4] A. Asokkumar, *Derivations in hyperrings and prime hyperrings*, Iran. J. Math. Sci. Inform., **8** (2013), 1-13.

[5] M. Bakhshi and R. A. Borzooei, *Ordered polygroups*, Ratio Mathematica, **24** (2013), 31-40.

[6] T. Changphas and B. Davvaz, *Properties of hyperideals in ordered semihypergroups*, Italian J. Pure Appl. Math., **33** (2014), 425-432.

[7] J. Chvalina, *Commutative hypergroups in the sence of Marty and ordered sets*, General algebra and ordered sets (Horni Lipova, 1994), 19-30.

[8] J. Chvalina and J. Moucka, *Hypergroups determined by orderings with regular endomorphism monoids*, Ital. J. Pure Appl. Math., **16** (2004), 227-242.

[9] I. Cristea and S. Jancic-Rašovic, *Composition hyperrings*, An. Stiint. Univ. "Ovidius" Constanta Ser. Mat., **21(2)** (2013), 81-94.

[10] B. Davvaz, *Isomorphism theorems of hyperrings*, Indian J. Pure Appl. Math., **35(3)** (2004), 321-331.

[11] B. Davvaz, P. Corsini and T. Changphas, *Relationship between ordered semihypergroups and ordered semigroups by using pseuoorders*, European J. Combinatorics, **44** (2015), 208-217.

[12] B. Davvaz and V. Leoreanu-Fotea, *Hyperring Theory and Applications*, International Academic Press, USA, 2007.

[13] D. Heidari and B. Davvaz, *On ordered hyperstructures*, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys., **73(2)** (2011), 85-96.

[14] S. Hoskova, *Quasi-order hypergroups determined by T-hypergroups*, Ratio Mathematica, **32** (2017), 37-44.

[15] S. Hoskova, *Upper order hypergroups as a reflective subcategory of subquasiorder hypergroups*, Ital. J. Pure Appl. Math., **20** (2006), 215-222.

[16] N. Kehayopulu and M. Tsingelis, *Pseudoorder in ordered semigroups*, Semigroup Forum, **50** (1995), 389-392.

[17] M. Krasner, *A class of hyperrings and hyperfields*, International J. Math. and Math. Sci., **6** (1983), 307-312.

[18] S. Mirvakili and B. Davvaz, *Applications of the α^*-relation to Krasner hyperrings*, J. Algebra, **362** (2012), 145-156.

[19] S. Mirvakili and B. Davvaz, *Relations on Krasner (m, n)-hyperrings*, European J. Combin., **31** (2010), 790-802.

[20] J. Mittas, *Hypergroups canoniques*, Mathematica Balkanica, **2** (1972), 165-179.

[21] J. Mittas, *Sur les hyperanneaux et les hypercorps*, Math. Balkanica, **3** (1973), 368-382.

[22] T. Vougiouklis, *The fundamental relation in hyperrings. The general hyperfield*, Algebraic hyperstructures and applications (Xanthi, 1990), 203–211, World Sci. Publ., Teaneck, NJ, 1991.

[23] T. Vougiouklis, *Hyperstructures and Their Representations*, Hadronic Press, Inc 115 Palm Harber, USA, 1994.

SABER OMIDI,
Department of Mathematics,
Yazd University,
Yazd, Iran.

BIJAN DAVVAZ,
Department of Mathematics,
Yazd University,
Yazd, Iran.
Email: davvaz@yazd.ac.ir, bdavvaz@yahoo.com